The linear function that fits the graph of f^{-1} is

$$y = 2.5x + 0.5 \qquad \text{slope} = 2.5,\ y\text{-intercept} = 0.5$$

If you know the equation of a function, you can transform it algebraically to find the equation of the inverse relation by first interchanging the variables.

Function: $y = 0.4x - 0.2$

Inverse: $x = 0.4y - 0.2$

The equation of the inverse relation can be solved for y in terms of x.

$$x = 0.4y - 0.2$$

$$y = 2.5x + 0.5 \qquad \text{Solve for } y \text{ in terms of } x.$$

To distinguish between the function and its inverse, you can write

$$f(x) = 0.4x - 0.2 \qquad \text{and} \qquad f^{-1}(x) = 2.5x + 0.5$$

Bear in mind that the x used as the input for function f is not the same as the x used as the input for function f^{-1}. One is time, and the other is distance.

An interesting thing happens if you take the *composition* of a function and its inverse. In the highway stripe example,

$$f(4) = 1.4 \qquad \text{and} \qquad f^{-1}(1.4) = 4$$

$$\therefore f^{-1}(f(4)) = 4$$

You get the original input, 4, back again. This result should not be surprising to you. The composite function $f^{-1}(f(4))$ means "How many hours does it take the crew to paint the distance it can paint in four hours?" There is a similar meaning for $f(f^{-1}(x))$. For instance,

$$f(f^{-1}(1.4)) = f(4) = 1.4$$

In this case 1.4 is the original input of the *inside* function.

Invertibility and the Domain of an Inverse Relation

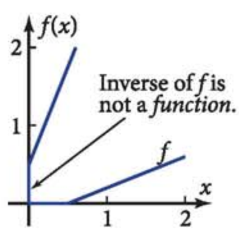

Figure 1-5d

In the highway stripe example, the length of stripe painted during the first half hour was zero because it took some time at the beginning of the shift for the crew to divert traffic and prepare the equipment. The graph of function f in Figure 1-5d includes times at the beginning of the shift, along with its inverse relation.